Energy harvesting in a pendulum system with impacts

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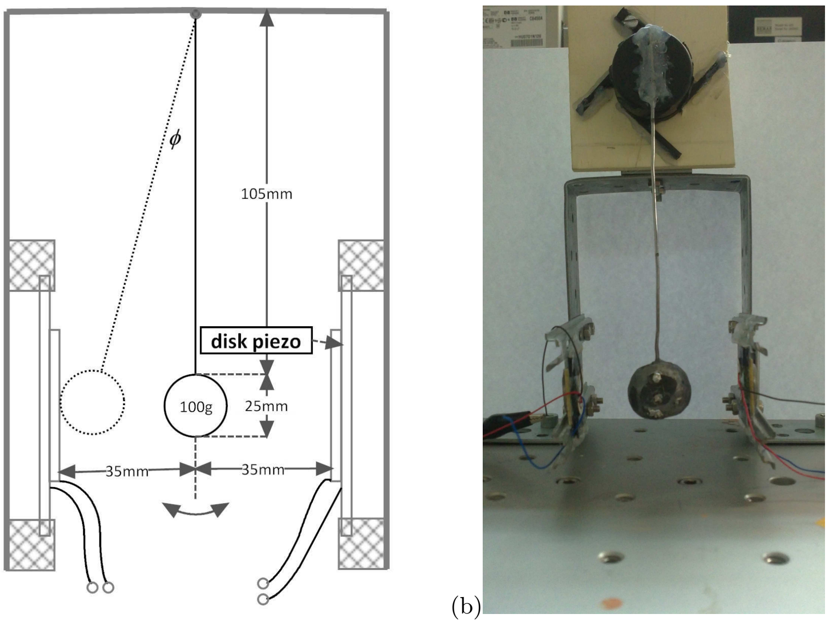
Abstract: We examine a simple nonlinear harvester system with an impacting pendulum. The mechanical vibration energy loss and its transduction to electrical power was modelled via a restitution coefficient. The system is motivated by the application to sea conditions monitoring. The pendulum device is tested to accumulated and/or scavenged energy of the see waves and water flows, and vortices appearance. The pendulum system is designed to be placed inside a small moving box in dry conditions.

1. Introduction

Recently, a considerable attention has been places in the electro-mechanical energy harvesters which can transform energy from ambient vibrations to electrical power through the mechanical resonator and piezoceramic transducers [1]. Non-linearities in vibrational energy harvesting systems are used to improve efficiency in the changing ambient conditions. The region of driving ambient vibration frequency can be broadened by nonlinear inclination of the resonance curve and the existence of additional solutions [2-3]. There appeared many proposals how to include nonlinearities into the energy harvesting systems [4] including the potentials modified by electrical, magnetic [5], or gravitational fields [6]. Among various approaches, impact phenomena were also investigated [7-10]. It is worth no notice that interesting results regarding dynamics of coupled pendulums with impacts were presented in [11-13]. In this paper we would like to continue this approach by modelling the impact between the pendulum and piezoelectric disks [14].

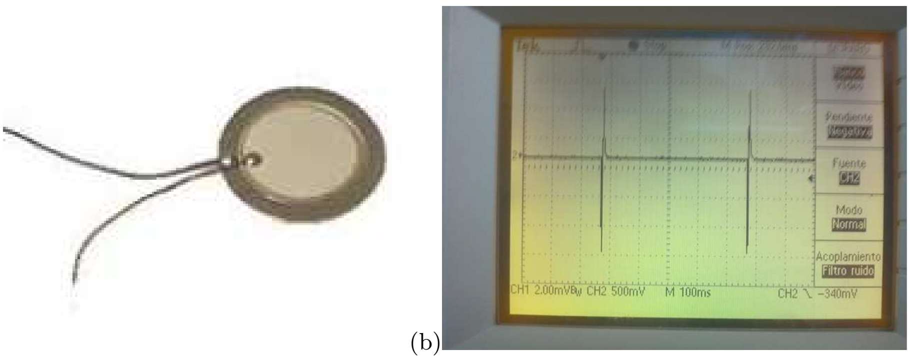
2. Model and results

The mathematical model of the pendulum excited vertically with a harmonic moving frame and impacting in the disk can be written as follows



(a) (b)

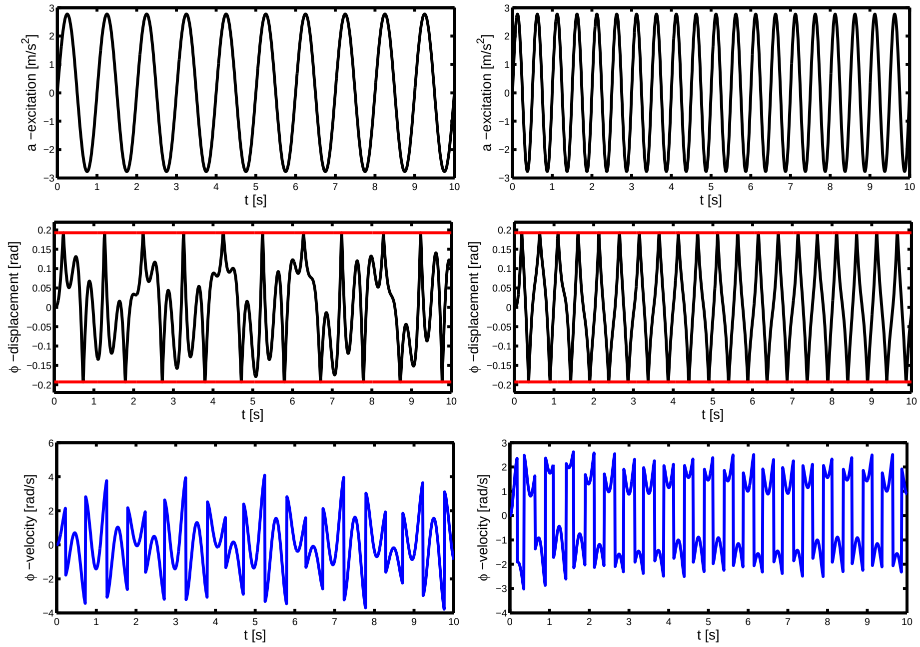
Figure 1. Schema of the system with corresponding lengths and concentrated mass swinging and impacting lead ball (a). Photo of the experimental stand (b),



(a) (b)

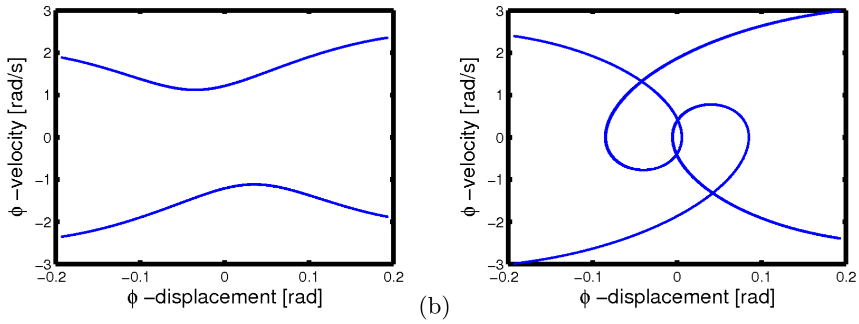
Figure 2. Piezoelectric disk used in the experiments (a). Output voltage measured on the piezoelectric disk (b).

$$
\begin{aligned}
&\ddot{\phi}_n + \frac{g - acos(\Omega t)}{l} \sin\phi = 0, && \text{for} \quad |\phi_n| < \phi_{max} \quad \text{(no impact)} \\
&\dot{\phi}_{n+1} = -k\dot{\phi}_n, && \text{for} \quad |\phi_n| = \phi_{max} \quad \text{(at the impact)}
\end{aligned}
\qquad (1)
$$



(a) (b) (c)

Figure 3. Fig. 3 Calculated angular displacement (b) and corresponding (c) velocity as the system response to harmonic excitation (a) (system parameters: k =0.8, $\Omega/2\pi$ =1Hz and 2Hz for left and right panes, respectively). Horizontal red curves (in Fig. 3b) denote limits defined by the maximal angle ϕ_{max}.



(a) (b)

Figure 4. Phase portraits for both frequencies $\Omega/2\pi$ =1Hz (a) and 2Hz (b)

where ψ and the swinging angle, g = 9.81m/s^2 is the gravity acceleration constant, l = 0.1175m length of the pendulum, a is the acceleration, excitation amplitude, k = 0.8 is the effective restitution coefficient dependent on the material properties. ϕ_{max} = 0.1927rad=11.04 deg is the limiting angle of contact between the pendulum and the piezoceramic disk (see the system geometry and experimental stand Figs. 1 and 2).

The results of numerical simulations are presented in Figs. 3 and 4. In particular, Figs. 3a-c show the calculated angular displacement (b) and corresponding velocity (c) as the system response to harmonic excitation (a) (system parameters: k =0.8, $\Omega/2\pi$ =1Hz and 2Hz for left and right panes, respectively.

One can easily see the difference in the response. The lower excitation frequency case (left panel in Fig. 3a-c) corresponds to the complicated oscillations composed with impacts and fairly small angle displacements while the case with higher excitation frequency represents the simple regular response.

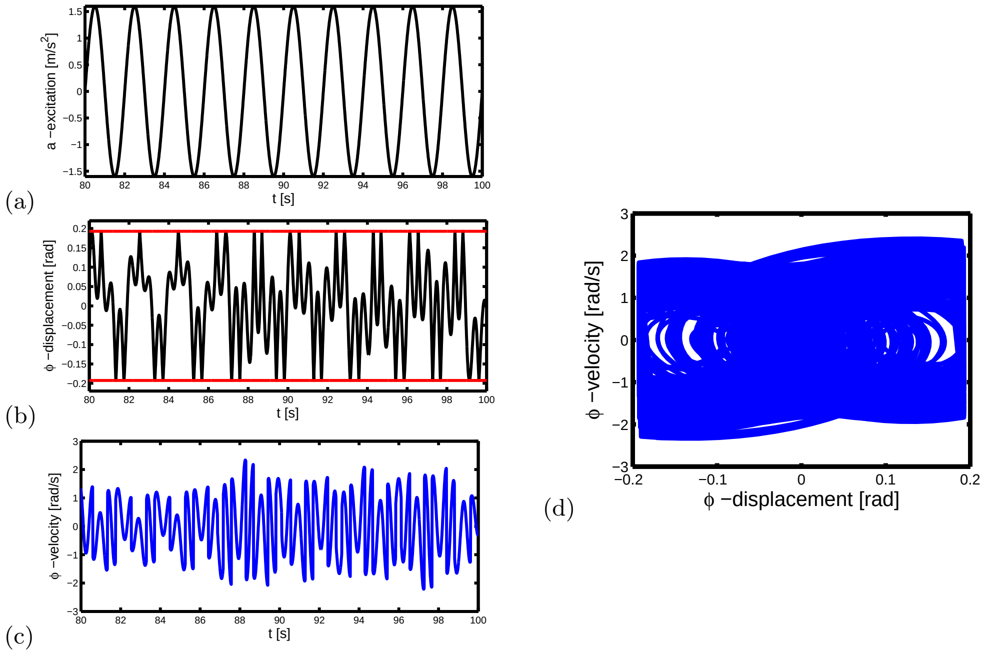


Figure 5. Chaotic system response of the system: the kinematical excitation (a), displacement (b), and corresponding velocity (c) (system parameters: a = 1.6 m/s^2, k =0.8, $\Omega/2\pi$ =0.5 Hz). Horizontal red curves (in Fig. 5b) denote limits defined by the maximal displacement angle.

It appeared that the both responses are regular as they are characterized the regular types of phase portraits (Fig. 4) as expected for any nonlinear system.

Additionally, some examples of the experimental results are presented in Tab. 1. In this table we report the response of the peak output voltage and relaxation time of the piezoceramic dicks after impacts. Interestingly, the system go through the chaotic response.

Table 1. Experimental results of output peak voltage and the relaxation time of the piezoceramic dicks.

excit. freq.	frame accell.	volt. peak	time inter.
[Hz]	amp. [g]	[mV]	[ms]
1	0.21	100.0	1.7
1	0.25	128.0	2.0
1	0.28	80.0	1.5
2	0.21	86.0	1.5
2	0.25	100.0	1.5
2	0.28	114.0	1.75
2	0.35	126.0	2.5
2	0.42	192.0	2.0

as was shown in Fig. 5. The nonperiodic response of the angle and angular velocities Figs. 5b and c is also reflected in the phase portrait Fig. 5d.

3. Conclusions

Our impacting device could be used on hermetically closed container in the sea to avoid influences of bio-deposits. Due to the impacts it has a broad-band energy characteristics which enables effectively harvest energy in variable conditions. Changing excitation frequency we observed bifurcations of the system response including the chaotic behaviour. However, to tell more about the output velocity or electrical power dependence on the type of solution, the corresponding bifurcation diagram of our system need to be studied more systematically.

Our composite disk were prepared in such a way to avoid the direct damage which could caused by impacts.

Acknowledgments

This research was supported by the Polish National Science Center (G.L. and A.R.) under the grant agreement No. 2012/05/B/ST8/00080.

References

1. S.P. Beeby, M.J. Tudor, N.M. White,: Energy harvesting vibration sources for microsystems applications, *Measurement Science and Technology*, 17, 2006, R175-R195.
2. P.D. Mitcheson, E.M. Yeatman, G.K. Rao, A.S. Holmes, T.C. Green,: Energy harvesting from human and machine motion for wireless electronic devices, *Proc. IEEE*, 96, 2008,

1457-1486.
3. A. Erturk, D. Inman,: *Piezoelectric Energy Harvesting.*, Chichester: John Wiley & Sons Ltd. 2011.
4. R.L. Harne, K.W. Wang: A review of the recent research on vibration energy harvesting via bistable systems, *Smart Mater. Struct.*, 22, 2012, 023001.
5. C.Y.K. Chee, L. Tong L, G.P. Steven: A review on the modeling of piezoelectric sensors and actuators incorporated in intelligent structures, *J. Intellig. Mat. Syst. Struct.*, 9, 1998, 3-19.
6. M.I. Friswell, S.F. Ali, S. Adhikari, A.W. Lees, O. Bilgen, G. Litak: Nonlinear piezoelectric vibration energy harvesting from a vertical cantilever beam with tip mass, *J. Intellig. Mat. Syst. Struct.*, 23, 2012, 1505-1521.
7. L-C. J. Blystad, E Halvorsen,: A piezoelectric energy harvester with a mechanical end stop on one side, *Microsyst. Technol.*, 17, 2011, 505-511.
8. L. Gu, C Livermore,: Impact-driven, frequency up-converting coupled vibration energy harvesting device for low frequency operation, *Smart Mater. Struct.*, 20, 2011, 045004.
9. C.P. Le, E. Halvorsen, O. Sorasen, E.M. Yeatman,: Microscale electrostatic energy harvester using internal impacts, *J. Intellig. Mat. Syst. Struct.*, 23, 2012, 1409.
10. M.S.M. Soliman, E.M. Abdel-Rahman, E.F. El-Saadany, R.R. Mansour: A wideband vibration-based energy harvester, *J. Micromech. Microeng.*, 18, 2008, 115021.
11. J. Awrejcewicz, G. Kudra, C.-H. Lamarque: Dynamics investigation of three coupled rods with a horizontal barrier, *Meccanica*, 38, 2003, 687-698.
12. J. Awrejcewicz, G. Kudra, C.-H. Lamarque: Investigation of triple pendulum with impacts using fundamental solution matrices, *Int. J. Bifurcation and Chaos*, 14, 2004, 4191-4213.
13. J. Awrejcewicz, G. Kudra: The piston-connecting rod-crankshaft system as a triple physical pendulum with impacts, *Int. J. Bifurcation and Chaos*, 15, 2005, 2207-2226.
14. C. Vinolo, D. Toma, A. Manuel, J. del Rio: An ocean kinetic energy converter for low-power applications using piezoelectric disk elements, *Eur. Phys. J. Special Topics*, 222, 2013, 1685-1698.

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