# Chapter 18 Dynamics of composite milling: application of recurrence plots to Huang experimental modes

G. Litak, R. Rusinek, K. Kecik, A. Rysak, A. Syta

Abstract We study the dynamics of a milling process of a composite material basing on the experimental time series of cutting force components measured in the feeding direction. By using the recurrence plots we observe the differences in the response of the system depending on the feeding direction with respect to composite fibers orientation. This effect has been found after decomposition on the Huang experimental modes. Showing the results of recurrences in particular experimental modes we advocate to use this quantity to analyze the stability of the cutting of composites. The difference between different cases was also noticed using Fourier transform and statistical parameters as RMS and kurtosis but for these methods the necessary time interval of the examined time series has to be much longer, while recurrence approach is designed for shorter time series.

## 18.1 Introduction

The cutting process is a basic technology to get the desired shape and surface parameters. In some conditions may be affected by vibration types of "chatter" as manifested in unexpected waves on the machined surface of the workpiece. This effect was noticed and described by Taylor in the early twentieth century [1]. However first attempts to explain this phenomenon took place 50 years after its discovery. The sources of these vibrations was seen in a number of non-linear deterministic effects, which include the mechanisms of self-excited vibration generation [2], the effects of regenerative cutting [3], the structural dynamics of the process [4, 5] and finally dry friction [6, 7].

Faculty of Mechanical Engineering, Lublin University of Technology, Nadbystrzycka 36, PL-20-618 Lublin, Poland e-mail: g.litak@pollub.pl, e-mail: r.rusinek@pollub.pl, e-mail: k.kecik@pollub.pl, e-mail: rysak.andrzej@gmail.com, e-mail: a.syta@pollub.pl

It should also be noted that these effects are not mutually exclusive. As a result, the elimination of vibration and stabilize cutting accompanying met with great interest in science and technology [8, 9, 10]. Short time-series studies have become important to understand the process and develop a better control strategy [11].

Recently, dynamics of milling process have been investigated intensively. The authors of papers [10, 12, 13, 14, 15, 16] focused on stability of milling process, bifurcations leading to chatter vibrations, and finally on identification of various types of system vibrations using nonlinear methods.

Resistance of fibers in the composites and possible damage mechanisms (such as fiber pullout, fiber fragmentation and delamination, matrix burning, and/or cracking), influence on the surface quality of a machinging process [17, 18, 19].

#### 18.2 Experimental setup and measured time series



Fig. 18.1 Photo of an experimental stand (a) and a schematic plot (b) of milling process configuration. Note, the angle  $\beta$  denotes the milling direction with respect to composite fibers (Tab. 18.1).

Milling process of composite material photo and schematics are presented in Fig. 18.1. In climb thread cutting process a finger cutter (full circuit milling) without colling. The parameters of the investigated milling process are shown in Tab. 18.1. In the experiment, we sampled values feed component forces  $F_x$  (Fig. 18.1) with frequency of 10kHz. For tests we used Carbon-fiberreinforced polymer CFRP based on unidirectional Carbon-epoxyde prepreg (Hexcel) with carbon fiber -AS7J 12K and epoxide resin M12.

The measured time series for the force  $F_x$  for fiber orientation angle  $\beta$  are presented in Fig. 18.2. The angle  $\beta$  was chosen  $\beta = 90^{\circ}$  and  $\beta = 75^{\circ}$  for D1 and D2, respectively.



Fig. 18.2 Time series of measured Force  $F_x$  (feed direction in Fig. 18.1). Note, the difference of scale in vertical axes.



Fig. 18.3 Fourier analysis of measured  $F_x$  time series: frequency response.

**Table 18.1** Milling process parameters. The angle  $\beta$  ( $\beta = 90^{\circ}$  for D1 and  $\beta = 75^{\circ}$  for D2) was denoted in Fig. 18.1b).

No.	cutting depth	feed ratio	feed ratio	milling width	rot. speed	cutting speed	angle $\beta$
measur.	[mm]	[mm/rev.]	[mm/min]	[mm]	[rpm]	[m/rev.]	[deg]
(D1)	0.8	0.0625	1500	12	12000	452,16	90
(D2)	0.8	0.0625	1500	12	12000	452,16	75

# 18.3 Analysis of the experimental response

The corresponding frequency spectrum is shown in Fig. 18.3. One can see the characteristic frequency of 200 Hz cutter rotation and its multiples. In these two cases, other components are dominant. If D2 is the frequency of 400 Hz, while in the case of D1 is 200 Hz. Note that the appearance multiples of 200 Hz implies nonlinear dynamics.



Fig. 18.4 Huang experimental mode (EM) decomposition (of measured  $F_x$ ). Consecutive modes 1-6 from bottom to top of figures.



Fig. 18.5 Recurrence rate *RR* versus the threshold value *e*.

**Table 18.2** Summary of statistics and recurrences for the milling process  $F_x$  component: number of experimental modes (No. EM), root mean square (RMS), kurtosis, recurrence rate (*RR*). Note RMS and kurtosis have been calculated for intervals of 1 s shown in Fig. 18.2. while recurrence for the first 0.2 s of the corresponding time series (see Figs. 18.2 and 18.6).

No. EM	RMS (D1)	RMS (D2)	kurtosis	kurtosis	RR (D1)	RR (D2)
	[N]	[N]	(D1)	(D2)	e = 0.8	e = 0.8
1	73.33	182.35	2.608	3.117	0.268	0.260
2	37.08	154.91	2.299	3.454	0.141	0.159
3	106.39	280.21	1.730	4.159	0.166	0.145
4	141.34	308.66	1.586	2.651	0.178	0.141
5	69.67	13.20	2.053	1.929	0.243	0.140

It also appears that in the case of D1 the spectrum are also showing additional structure of less than 200 Hz, which may be associated with the

18 Dynamics of composite milling



Fig. 18.6 Recurrence plots for e = 0.8 in two cases D1 (a,c,e,g,i) and D2 (b,d,f,h,j) for particular experimental modes. Note that, the plot is made using the Eq. 18.3.

orientation of composite fibers. Namely, this is the most transparent difference in the spectra of these two cases.

## 18.4 Recurrence plots for Huang experimental modes

In the analysis by Hilbert-Huang one performs the so-called signal decomposition into experimental modes (Huang decomposition):  $F_x^1(t)$ ,  $F_x^2(t)$ , ...,  $F_x^m(t)$  [20, 21]:

$$F_x(t) = \sum_{j=1}^m F_x^j(t) + r_m,$$
(18.1)



Fig. 18.6 Continuation.

where  $r_m$  is a truncation error. Each next experimental j mode is defined after subtracting average of maximum and minimum values interpolated by a cubic splines of the local envelope  $F_x^{j-1}(t)$ . Note that the first mode  $F_x^1(t)$  is obtained from the original signal  $F_x(t) = F_x^0(t)$  and the Huang decomposition procedure.

The first 6 Huang modes obtained using the above schema are plotted in Fig. 18.4. One can see that the amplitude reach maximum for mode 4, which could be the most important to distinguish the type of vibrations.

In the next step we provide the second coordinate as the numerical derivative  $F_x^{\prime i}(t)$  for each mode  $F_x^i(t)$ . After normalization of each two variables  $\tilde{\mathbf{F}}_x^i(t) = (\tilde{F}_x^{\prime i}(t), \tilde{F}_x^i(t))$  for given mode trough the corresponding standard deviations we get phase vector representation

$$\tilde{\mathbf{F}}_x^1(t), \tilde{\mathbf{F}}_x^2(t), \dots, \tilde{\mathbf{F}}_x^m(t),$$
(18.2)

18 Dynamics of composite milling



Fig. 18.6 Continuation.

where m is a natural number of the highest mode truncation (in our case m = 6).

Using such formulation we performed recurrence analysis for each mode to compare D1 and D2 time series (Fig. 18.2). The recurrence rate (RR)parameter can be defined for each mode separately. It is defined as a fraction of off diagonal  $i \neq j$  following inequality [22]

$$|\tilde{\mathbf{F}}_x^n(t_i) - \tilde{\mathbf{F}}_x^n(t_j)| < e, \tag{18.3}$$

where e is the given threshold number.

Namely, RR reads

$$RR = \frac{1}{N(N-1)} \sum_{ij} \theta(e - |\tilde{\mathbf{F}}_x^n(t_i) - \tilde{\mathbf{F}}_x^n(t_j)|) \quad \text{(for } i \neq j\text{)}, \quad (18.4)$$

where  $\theta(.)$  defines the Heaviside step function, N denotes the length of considered time series, while n indicates the mode. Note that RR has been already proposed as a good quantity to distinguish some different responses of dynamical systems [23, 24, 25]. In this paper we adopt this idea. The results for the first 5 corresponding modes (see Fig. 18.5) have been used for calculations of RR One can clearly see the difference in modes functions versus threshold value e. The most prominent difference is expressed in mode 4 and 5 and also in modes 1-3 intersection behaviour in Fig. 18.5 D1 in contrary to the separate (no-crossing) grow tendency in Fig. 18.5 D2. These behaviours have the origin in different mode vibrations for D1 and D2 time series. For better clarity we have also plotted the corresponding recurence plots (Fig. 18.6 a-j). One can clearly see different patterns in particular RP figures. The most regular is Fig. 18.6g (for D1) where the diagonal long lines are most repeatable. This opposes to Fig. 18.6h (for D2) where the lines have the fairly shorter lengths. This results, and also other figures (from the set of figures: Figs. 18.6a-j) in a smaller extend, imply that the time series D1 are more periodic than D2. Note that this conclusion can be draw from fairly short time series. Interestingly in that case (D1) the cutter feed direction is oriented perpendicularly to the composite fibers.

The modal results helped to capture visible changes in the statistical measures (Tab. 18.2). They can be also used to the design of improved control algorithm milling. Note the nonmonotonic evolution of RR (Tab. 18.2) in case of the D1 case. Fairly larger kurtosis in D2 case (see mode 3 in Tab. 18.2) implies intermittency confirmed also by RP figures (see the difference between Figs. 18.6e & f, also Figs. 18.6g & h, and Figs. 18.6i & j). It seems that the configuration of the angle  $\beta$  of the D2 case introduces an additional fairly low frequency modulation in the case of D1 increases vibration in the whole range. The modulation effect of D2 is more visible in the frequency spectrum Fig. 18.3a.

#### 18.5 Conclusions

The results of measurement and analysis of signals based on multi-resolution method for experimental modes. Unlike the Fourier transform, it is applied to non-stationary signals as well as those that exhibit the phenomenon of intermittency. In our case, we have examined the process of composite milling tools with different orientations relative to the direction of the fibers.

Recurences inform about a specific modulation and may also indicate a non-linear nature of these oscillations. However, to provide specific guidance and make a more systematic study. Some more information about the nature of identified vibrations can be learned from other parameters which are in use in recurence quantification analysis [22]. However repeating the procedure for other parameters in the adopted processing conditions goes beyond this paper and will be reported in a separate article.

### Acknowledgment

The financial support of Structural Funds in the Operational Programme– Innovative Economy (IE OP) financed from the European Regional Development Fund–Project "Modern material technologies in aerospace industry", No. POIG.01.01.02-00-015/08-00 is gratefully acknowledged. 18 Dynamics of composite milling

#### References

- 1. Taylor, F.: On the art of cutting metals. Trans. ASME 28, 310–350 (1907).
- Arnold, R.N.: The mechanism of tool vibration in the cutting of steel. Proc. Inst. Mech. Eng. 154, 261–284 (1946).
- Tobias, S.A., Fishwick, W.: A Theory of Regenerative Chatter. The Engineer, London (1958).
- Tlusty, J., Polacek, M.: The stability of machine tool against self-excited vibrations in machining. ASME Int. Res. Prod. Eng. 465–474 (1963).
- Merrit, H.E.: Theory of self-excited machine-tool chatter, ASME J. Eng. Ind. 87, 447–454 (1965).
- Wu, D.W., Liu, C.R.: An analytical model of cutting dynamics. Part 1: Model building. ASME J. Eng. Ind. 107, 107–111 (1985).
- Wu, D.W., Liu, C.R.: An analytical model of cutting dynamics. Part 2: Verification. ASME J. Eng. Ind. 107, 112–118 (1985).
- Altintas, Y.: Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design. Cambridge University Press, Cambridge (2000).
- Warminski, J., Litak, G., Cartmell M.P., Khanin R., Wiercigroch W.: Approximate analytical solutions for primary chatter in the nonlinear metal cutting model. J. Sound Vibr. 259, 917–933 (2003).
- Insperger, T., Gradisek, J., Kalveram, M., Stepan, G., Winert K., Govekar E.: Machine tool chatter and surface location error in milling processes. J. Manufac. Sci. Eng. 128, 913-920 (2006).
- Ganguli, A., Deraemaeker, A., Preumont, A.: Regenerative chatter reduction by active damping control. J. Sound Vibr. 300, 847–862 (2007).
- Mann, B.P., Insperger, T., Bayly, P.V., Stepan, G.: Stability of up-milling and downmilling, part 2: experimental verification. Int. J. Mach. Tools Manuf. 43, 35-40 (2003).
- Mann, B.P., Bayly, P.V., Davies, M.A., Halley, J.E.: Limit cycles, bifurcations, and accuracy of the milling process. J. Sound Vib. 277, 31–48 (2004)
- Insperger, T., Mann, B.P., Surmann, T., Stepan, G.: On the chatter frequencies of milling processes with runout. Int. J. Mach. Tools Manuf. 48, 1081–1089 (2008).
- Litak, G., Syta, A., Rusinek, R.: Dynamical changes during composite milling: recurrence and multiscale entropy analysis, The Int. J. Adv. Manufact. Technol. 56, 445–453 (2011).
- Sen, A.K., Litak, G., Syta, A., Rusinek, R.: Intermittency and multiscale dynamics in milling of fiber reinforced composites, Meccanica 48, 738–789 (2013).
- Rao, G.V.G., Mahajan, P., Bhatnagar, N.: Micromechanical modeling of machining of FRP composites cutting force analysis. Compos. Sci. Technol. 67, 579–593 (2007).
- Koplev, A., Lystrup, A., Vorm, T.: The cutting process, chips, and cutting forces in machining CFRP. Composites 14, 371–376 (1983).
- Rusinek, R.: Cutting process of composite materials: an experimental study. Int. J. Non-Linear Mech. 45, 458–462 (2010).
- Huang, N.E., Shen, Z., Long, S.R., Wu, M.L.C., Shih, H.H., Zheng, Q.N., Yen, N.C., Tung, C.C., Liu, H.H.: The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, Proc. Roy. Soc. Lond. 454, 903–993 (1998).
- Litak, G., Kecik, K., Rusinek, R.: Cutting force response in milling of Inconel: Analysis by wavelet and Hilbert-Huang transforms Latin American Journal of Solids and Structures 10, 133–140 (2013).
- Marwan, N., Romano, M.C., Thiel, M., Kurths, J.: Recurrence plots for the analysis of complex systems Physics Rep. 438, 237-329 (2007).
- Litak, G., Sawicki, J.T., Kasperek, R.: Cracked rotor detection by recurrence plots, Nondestruc. Test. Eval. 24, 347–351 (2009).

- Litak, G., Wiercigroch, M., Horton, B.W., Xu, X.: Transient chaotic behaviour versus periodic motion of a parametric pendulum by recurrence plots, Zeit. Ang. Math. Mech. 90, 33–41 (2010).
- 25. Syta, A., Jonak, J., Jedlinski, L., Litak, G.: Failure diagnosis of a gear box by recurrences, J. Vib. Acous. **134**, 041006, (2012).

10